EE 341: Linear Systems Analysis
Assignment 4: The FFT

Due Date: Wednesday, 28 February 2001

When using a digital computer, frequency analysis means using a Fast Fourier Transform (FFT). This necessitates we spend some time becoming familiar with using the FFT to study the frequency content of a discrete-time signal.

1. **MATLAB function FFT**

   In this problem you will learn how to use the MATLAB command `fft`. First, use the `help` feature in MATLAB to learn the syntax of the `fft` function. The FFT function computes the Discrete Fourier Transform (DFT) of a sequence. In general the FFT of a sequence will be a complex function so you will need to look at the magnitude and phase separately. The MATLAB commands `abs` and `angle` are useful for obtaining the magnitude and phase of a complex valued sequence. Also, since the FFT only has values at discrete frequencies, it may be useful to do the plots with `stem` to reinforce that idea, but continuous frequency plots (i.e. using `plot`) are often used since they are closer to the DTFT that you are ultimately interested in.

   The FFT outputs a sequence that corresponds to the range $0 \le \omega \le 2\pi$ (or $0 \le \bar{f} < 1$ for normalized frequency $\bar{f}$). You are probably more familiar with seeing the spectrum plotted over the range $-\pi \le \omega < \pi$ (or $-0.5 \le \bar{f} < 0.5$, as in lab 3). The `fftshift` function can be used for this purpose.

   Plot magnitude of the FFT of the following signal before and after the `fftshift`

   $$x[n] = 1 + \cos(2\pi f n); \;\; 0 \le n \le 127$$

   for the cases where $f = 0.4$ and $f = 0.65$. Use your understanding of the relation between discrete and continuous time to plot the magnitude of the Fourier Transform of the continuous time signal that these correspond to, assuming the sampling period is $T = 10^{-4}$. Be sure to label the frequency axis correctly and indicate whether you are plotting in radians or Hertz or normalized frequency.
   <u>Turn in a 3-part plot for each signal: unshifted DFT, shifted DFT, and the shifted DFT with a Hz frequency scale. Discuss why the frequency peak locations make sense.</u>

2. **Frequency Shifting**

   For each of the following sequences, let $f_1 = 0.2$ and $0 \le n \le 255$. The function `sinc.m` generates the sinc function – get it from the class web page. (Note that MATLAB also has a sinc function, but it has a slightly different form, so please use this one to avoid grading confusion.) Plot the magnitude and phase plots (using plot), where the magnitude and phase plots are over the range $-0.5 \le \omega < 0.5$ (normalized frequency).

   (a) $x[n] = \text{sinc}(f_1(n - 64))$.

   (b) $x[n] = \text{sinc}(f_1(n - 64))(-1)^n$.

   (c) $x[n] = \text{sinc}(f_1(n - 64)) \cos(2\pi f_2 n)$ where $f_2 = 0.3$.

(d) $x[n] = \text{sinc}(f_3(n-64))\cos(2\pi f_3 n)$ where $f_3 = 0.4$.

What type of signals are these (low pass, high pass, etc.)? Turn in the plots for (d). Explain why (d) does not have a flat frequency response in the passband.

3. **DTFT vs. DFT**

In this problem you will observe the errors that are produced when applying the DFT to infinite duration sequences.

(a) **Effect of frequency sampling**
Let $h[n] = a^n u[n]$ with $a = 0.75$. Its DTFT is: $H(\Omega) = \frac{1}{1-ae^{-j\Omega}}$. Generate the 16-point DFT sequence $H[k]$ by evaluating $H(\Omega)$ at the 16 appropriate frequency points (not by using the FFT). Form $h_{I16}[n]$, the 16-point IDFT of $H[k]$ and the error sequence, $e_{I16}[n] = h[n] - h_{I16}[n]$. Repeat for $N = 32$. Turn in a plot of $h_{I16}[n]$ and $e_{I16}[n]$ on one page using subplot. Repeat for $N = 32$. Describe the differences between the inverse DFTs of the 16-point and 32-point cases.

(b) **Effect of finite time window** Note that in problem 1 you don't get a perfect impulse at the desired frequency (as you would expect for the DTFT). This is because the DTFT is based on a finite-length piece of the signal. To understand this better, generate the signal

$$x[n] = 1 + \cos(2\pi f n); \; f = 0.4$$

for two different lengths: $0 \le n \le 127$ and $0 \le n \le 31$ Using `plot`, plot the magnitude of the FFT of both signals, using a 128-length FFT in both cases. Turn in the magnitude plots. Discuss what the time truncation corresponds to in the frequency domain that would explain the plot differences.