Name _____

ID Number _____

Section _____

Midterm – EE341

6 February 2001

Instructions:

- The test is closed book and you are allowed one 8.5×11 page of notes.
- **Show all work.** Partial credit will be given for partial work; NO credit will be given for no work even if the answer is correct.
- Be sure to state all assumptions made and check them when possible.
- There are four problems on six pages, including the cover and formula sheet. A blank page is attached at the end in case you need extra space.

Honor Code:
This exam represents only my own work. I did not give or receive help on this exam.

_____ _____

Signature Date

Formula Sheet

- **Complex numbers and sinusoids:** $(j = \sqrt{-1})$

$$z = a + jb = re^{j\theta} \quad \Rightarrow \quad |z| = r = \sqrt{(a^2 + b^2)};\ \angle z = \theta = \arctan(\frac{b}{a})$$
$$Re\{z\} = a = r\cos\theta;\ Im\{z\} = b = r\sin\theta$$

$$\cos(\phi) = \frac{1}{2}(e^{j\phi} + e^{-j\phi}) \qquad \sin(\phi) = \frac{1}{2j}(e^{j\phi} - e^{-j\phi}) \qquad e^{j\phi} = \cos(\phi) + j\sin(\phi)$$
$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) \qquad \sin(\phi) = \cos(\phi - \pi/2)$$

- **Summation formulas**

$$\sum_{k=n}^{\infty} a^k = \frac{a^n}{1-a} \text{ for } |a| < 1, n \geq 0 \qquad \sum_{k=0}^{\infty} ka^k = \frac{a}{(1-a)^2} \text{ for } |a| < 1 \qquad \sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a} \text{ for } a \neq 1$$

- **Convolution equations and properties**

$$y[n] = x[n] * h[n] \quad = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$$

  - Commutative: $x[n] * h[n] = h[n] * x[n]$
  - Associative: $(x[n] * h_1[n]) * h_2[n] = x[n] * (h_1[n] * h_2[n])$
  - Distributive: $x[n] * (h_1[n] + h_2[n]) = (x[n] * h_1[n]) + (x[n] * h_2[n])$

- **Properties of the unit impulse function:**

$$x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0] \qquad x[n_0] = \sum_{k=-\infty}^{\infty} x[n]\delta[n-n_0] \qquad x[n-n_0] = x[n] * \delta[n-n_0]$$

$$u[n] = \sum_{k=-\infty}^{n} \delta[k] \qquad \delta[n] = u[n] - u[n-1]$$

- **Eigenfunction property of LTI systems:**

$$x[n] = Aa^n \quad \rightarrow \boxed{H(z)} \rightarrow \quad y[n] = AH(a)a^n$$
$$x[n] = A\cos(\Omega_0 n + \phi) \quad \rightarrow \boxed{H(z)} \rightarrow \quad y[n] = A|H(e^{j\Omega_0})|\cos(\Omega_0 n + \phi + \angle H(e^{j\Omega_0}))$$

- **LTI system transfer function:**

$$\sum_{k=0}^{n} a_k y[n-k] = \sum_{l=0}^{m} b_l x[n-l] \qquad \Longleftrightarrow \qquad H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{l=0}^{m} b_l z^{-l}}{\sum_{k=0}^{n} a_k z^{-k}}$$
$$h[n] \qquad \Longleftrightarrow \qquad H(z) = \sum_{-\infty}^{\infty} h[n]z^{-n}$$

1. (15 points)
   For each of the two discrete-time signals below, determine if it is periodic and if so find the fundamental period.

   (a) (5 points) $x_1[n] = \cos(2n - 3)$

   (b) (10 points) $x_2[n] = e^{-j\frac{2\pi}{3}n} + \cos(\frac{3\pi}{4}n)$

2. (30 points)
Consider an LTI system with impulse response $h[n] = u[n] - u[n-3] - \delta[n+2]$

(a) (5 points) Sketch the impulse response.

(b) (5 points) Is the system causal? Explain.

(c) (5 points) Is the system stable? Explain.

(d) (15 points) Find and sketch the response of the system to $u[n]$, the unit step function.

3. (20 points)
From the six systems described below, find two that cannot be LTI and explain why not for each (e.g. "not linear because ..." OR "not TI because ..." OR "not both because ..."). [Note: There are at least two.]

$T_A: \ y[n] = nx[n] + x[n-1]$

$T_B: \ x[n] = 1 \rightarrow \boxed{T_B} \rightarrow y[n] = 0$

$T_C: \ y[n] = \sum_{k=-\infty}^{n-1} (-0.5)^k x[k]$

$T_D: \ x[n] = 1 \rightarrow \boxed{T_D} \rightarrow y[n] = (-1)^n$

$T_E: \ h[n] = 2^n u[n]$

$T_F: \ \delta[n] \rightarrow \boxed{T_F} \rightarrow u[n] \rightarrow \boxed{T_F} \rightarrow n^2 u[n]$

4. (35 points)
Each system on the left side is a causal LTI system. Draw a line from each of the systems on the left side to the system description on the right side that goes with it. ($H(z)$ is the transfer function, $y_c[n]$ is the natural response, $h[n]$ is the impulse response, and $s[n]$ is the step response.) On the left side, circle one system that is not stable. *Include a brief justification for EACH of your choices at the bottom of the page.*

$T_1: \ y[n] = x[n] - x[n-1]$

$T_2: \ y[n] - y[n-1] = x[n]$

$T_3: \ y[n] - y[n-3] = x[n]$

$$H(z) = \frac{1}{1 - z^{-1}}$$

$$y_c[n] = A + B\cos(\frac{2\pi}{3}n + \phi)$$

$$h[n] = (-1)^n u[n]$$

$$s[n] = u[n] - u[n-1]$$

End Of Exam